3-4 Activity

**3-4-1: Exponential Functions**

State what it means to be “growing exponentially”.

Give sequences: linear, exponential, other.: What did you do to make your determination? (active Reading: 3.1.8-9

Rate of change is multiple of itself (a^x table)

FM: 7.1 : Moore’s Law states that processor speeds, or overall processing power for computers will

double about every 18 months. Likewise, the cost to produce a comparable computer will be

cut in half.

What does this mean? To simplify the mathematics, let’s assume it takes 2 yrs to double/half instead of 18

months.

Example 1: In 1988, the number of transistors in the Intel 386 SX microprocessor was 275,000. What

was the approximate transistor count of the Pentium II Intel microprocessor in 1998?

Example 2: A personal computer that cost $3,000 in 2002 would cost about how much now?

The relative growth rate for bacteria from Sully’s tongue is 80% per hour after eating

a Turkish Pizza. Sully swabs his mouth and starts a culture that, 4 hours later, shows

approximately 50,000 bacteria. How many bacteria did Sully start the culture with? (Guess & Check)?

FM #9: The equation

y 25,000  .01 04 models the salary of an employee who receives an annual raise.

Give the meaning of each number and variable in this equation.

25,000: \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ 0.04:\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ 1:\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

A ball is dropped from a height of 12 feet and is allowed to bounce over and over. The height of each bounce is modeled in the exponential DECAY model below.

Bounce 0 1 2 3 4 --- 8 --- 100

Height (ft) 12 7.8 5.07 3.2955 2.142075 --- ??? --- ???

10. Function: \_\_\_\_\_\_\_\_\_\_\_\_\_\_ 11. 8th Bounce: \_\_\_\_\_\_\_\_\_\_\_\_ 12. 100th Bounce\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

S-Z; p. 420: The value of a car can be modeled by V (x) = 25 4 5 x , where x ≥ 0 is age of the car in years and V (x) is the value in thousands of dollars. 1. Find and interpret V (0). 2. Sketch the graph of y = V (x) using transformations. 3. Find and interpret the horizontal asymptote of the graph you found in

S-Z: p. 421: According to Newton’s Law of Cooling8 the temperature of coffee T (in degrees Fahrenheit) t minutes after it is served can be modeled by T(t) = 70 + 90e −0.1t . 1. Find and interpret T(0). 2. Sketch the graph of y = T(t) using transformations. 3. Find and interpret the horizontal asymptote of the graph.

S-Z: p. 485: #25: The Law of Uninhibited Growth also applies to situations where an animal is re-introduced into a suitable environment. Such a case is the reintroduction of wolves to Yellowstone National Park. According to the National Park Service, the wolf population in Yellowstone National Park was 52 in 1996 and 118 in 1999. Using these data, find a function of the form N(t) = N0e kt which models the number of wolves t years after 1996. (Use t = 0 to represent the year 1996. Also, round your value of k to four decimal places.) According to the model, how many wolves were in Yellowstone in 2002? (The recorded number is 272.) (Note: Give k for now)?

Active Reading: 3.1.10-12: A dosage rate of 250 mg/hr is reduced by 5 mg/hour each hour.

1. Complete the table for a dosage rate reduced by 5 mg/hour each hour.

|  |  |
| --- | --- |
| t (hours) | Dosage (mg/hr) |
| 0 |  |
| 1 |  |
| 2 |  |
| 3 |  |
| 4 |  |

What is the rate of change m between any two ordered pairs in the table?

A patient’s dosage rate of 250 mg/hr is reduced by 2% each hour.

1. What growth factor gives us a 2% decrease?

b=

1. Complete the table for a dosage reduced by 2% each hour (*round your answers to 2 decimal places*).
2. Consider again the 250 mg/hour dosage rate and reducing it by 2% each hour.

This time instead of the actual results, write the initial value followed by multiples of the growth factor for every hour that passes.

|  |  |
| --- | --- |
| t (hours) | Dosage (mg/hr) |
| 0 | 250⋅(0.98)0 |
| 1 | 250() |
| 2 | 250()() |
| 3 | 250()()() |
| 4 | 250()()()() |

Notice how the expressions in each output cell could be simpified by using exponents to count the repeated growth factor. Good to know!

1. 3.1.15-16: Is one decrease of 60 students the same as three decreases of 20 students? If we start with 300 students and subtract 60 students, how many students remain? 300−60= students.
2. If we start with 300 students and subtract 20 students three consecutive times, how many students remain?

300− − −

=300−3⋅

=

1. Is one decrease of 60 students the same as three decreases of 20 students?

yes  
 no

Is one decrease by 60%60% of students the same as three decreases by 20%20% of students? What is the growth factor associated with a 60% decrease? Multiply by to reduce by 60%.

If we start with 300 students then decrease enrollment by 60%, there will be students remaining.

Active Reading (gist) HW #19 An investment grows by 1.8% each year. By what percent will it increase after 5 years? Round your answer to the nearest hundredth of a percent.

HW #18 Find a formula for the exponential function V=h(t) that gives the value of an item initially worth $5000 that loses half its value every 4 years.

HW #20: If the side lengths of a square are each increasing by 6% per hour, by what percent will the *area* of the square increase after 5 hours? Give your answer to the nearest hundredth of a percent.

MFG: 4.1: HW #17 (thru #22): During a vigorous spraying program, the mosquito population was reduced to 3434 of its previous size every 22 weeks. If the mosquito population was originally estimated at 250,000,250,000, how many mosquitoes remained after 33 weeks of spraying? After 88 weeks?

1. MFG: 4.1 : HW #42: In 2006,2006, a new Ford Focus cost $15,574.$15,574. The value of a Focus decreases exponentially over time. Write a formula for the value of a Focus. (You do not know the value of the decay factor, b,b, yet.)
2. A 22-year old Focus cost $11,788.$11,788. Find the decay factor and the percent rate of depreciation, rounded to the nearest tenth of a percent.
3. About how much would a 44-year old Focus cost?

MFG: 4.1 HW #61: Francine says that if a population grew by 48%48% in 66 years, then it grew by 8%8% per year. Is she correct? Either justify or correct her calculation.

**3-4-2: Modeling with Exp. Functions revisited**

S-Z: 6.5: p. 471, 6.5.1: Example 6.5.1. Suppose $2000 is invested in an account which offers 7.125% compounded monthly. 1. Express the amount A in the account as a function of the term of the investment t in years. 2. How much is in the account after 5 years? 3. How long will it take for the initial investment to double? 4. Find and interpret the average rate of change5 of the amount in the account from the end of the fourth year to the end of the fifth year, and from the end of the thirty-fourth year to the end of the the 35th year.

S-Z 6.5 p. 482 HW #8: How much money needs to be invested now to obtain $2000 in 3 years if the interest rate in a savings account is 0.25%, compounded continuously? Round your answer to the nearest cent.

Active Reading: 3.2.3: Consider the expression

296(1.013)t/10

with t counting time in years.

1. What is the value of the exponent after 10 years?

The exponent is after 10 years.

1. What is the percent increase on the initial value after 10 years?

The increase is %.

1. How many times does this same percent increase occur in 20 years? times.
2. How many times does this same percent increase occur in 30 years? times.
3. Complete the following sentence:

The formula f(t)=296(1.013)t10 states an initial value of increases by % every years.

1. Consider a different formula f(t)=30(0.87)t/6 with t again time in years.

It states an initial value of

?

is constant

increases

decreases

none of the above

by % every years

Active Reading: 3.4.4: A certain pain relieving drug has a half-life of about 33 hours. If a person has this drug in their system, how long will it take for the drug to reduce to 10%10% of the drug in the body?

[Solution](https://www.mhcc.edu/precalc1/exponential-functions-gist.html)

We use an exponential model abtabt because of the repeated 50%50% loss. The problem states it takes 33 hours to reduce by 50%.50%. Therefore the hourly growth factor, bb occurs 33 times for a total of 50%50% loss. In math we say

b3=0.5b3=0.5

and solving for bb we get

b3=0.5(b3)1/3=(0.5)1/3b=(0.5)1/3.b3=0.5(b3)1/3=(0.5)1/3b=(0.5)1/3.

Because the problem does not tell us exactly how much of the drug was in the system to begin with, for simplicity we will imagine that beginning amount was 11mg. Now, knowing the hourly percent loss, we can write a formula

D(t)=abt=1((0.5)1/3)t=(0.5)t/3D(t)=abt=1((0.5)1/3)t=(0.5)t/3

where D(t)D(t) is the amount of drug in the body in mg and tt is time in hours.

We use the formula to determine how many hours it takes to reduce to 10% graphically.

Active Reading 3.4.23: For each function, determine the horizontal asymptote.

* g(x)=2^x has the horizontal asymptote:
* ?
* x
* y
* = h(x)=5(1.03)^x has the horizontal asymptote:

?

x

y

= k(x)=0.92^x+3 has the horizontal asymptote:

?

x

y

=

* m(x)=e^x has the horizontal asymptote:

?

x

y

=

Active Reading: 3.4.25: Decide if each expression below will *increase* x by 5%.

1. x+0.05?
2. ?
3. yes
4. no
5. 0.05x?
6. ?
7. yes
8. no
9. 0.5x?
10. ?
11. yes
12. no
13. x+0.05x?
14. ?
15. yes
16. no
17. 1.05x?
18. ?
19. yes
20. no

Decide if each expression below will *decrease* x by 5%.

1. 0.95x?
2. ?
3. yes
4. no
5. x−0.05?
6. ?
7. yes
8. no
9. −0.5x?
10. ?
11. yes
12. no
13. x−0.05x?
14. ?
15. yes
16. no
17. 0.05x?
18. ?
19. yes
20. no

Each function below represents the population of a different city, measured in thousands of people, where the input t is time in years.

A(t)=50(0.95)tB(t)=50(0.75)tC(t)=25(1.05)tD(t)=25(1.5)t

For each verbal description, choose the correct function:

* “The city begins with 25,000 people and increases by 5% per year.”

Answer:

?

A

B

C

D

* “The city begins with 50,000 and decreases by 5% per year.”

Answer:

?

A

B

C

D

* “The city begins with 25,000 and increases by 50% per year.”

Answer:

?

A

B

C

D

* “The city begins with 50,000 and decreases by 25% per year.”

Answer:

?

A

B

C

D

APC: p. 161 #4: In the year 2003, a total of 7.2 million passengers took a cruise vacation. The global cruise industry has been growing at 9% per year for the last decade. Assume that this growth rate continues. (a) Write a formula for to approximate the number, N, of cruise passengers (in millions) t years after 2003. (b) How many cruise passengers (in millions) are predicted in the year 2011? (c) How many cruise passengers (in millions) were there in the year 2000?

APC p. 161: #5: The populations, P, of six towns with time t in years are given by 1 P 800(0.78) t 2 P 900(1.06) t 3 P 1600(0.96) t 4 P 1400(1.187) t 5 P 500(1.14) t 6 P 2800(0.8) t Answer the following questions regarding the populations of the six towns above. (a) Which of the towns are growing? (b) Which of the towns are shrinking? (c) Which town is growing the fastest? What is the annual percentage growth RATE of that town? (d) Which town is shrinking the fastest? What is the annual percentage decay RATE of that town? (e) Which town has the largest initial population? (f) Which town has the smallest initial population?

APC: p. 163: #10 (HA: y=0): A cup of hot coffee is brought outside on a cold winter morning in Winnipeg, Manitoba, where the surrounding temperature is 0 degrees Fahrenheit. A temperature probe records the coffee’s temperature (in degrees Fahrenheit) every minute and generates the data shown in Table 3.1.13. t 0 2 4 6 8 10 F(t) 175 129.64 96.04 71.15 52.71 39.05 Table 3.1.13: The temperature, F, of the coffee at time t. a. Assume that the data in the table represents the overall trend of the behavior of F. Is F linear, exponential, or neither? Why? b. Is it possible to determine an exact formula for F? If yes, do so and justify your formula; if not, explain why not. c. What is the average rate of change of F on [4, 6]? Write a sentence that explains the practical meaning of this value in the context of the overall exercise. d. How do you think the data would appear if instead of being in a regular coffee cup, the coffee was contained in an insulated mug?

OR

APC p. 163 #11: (HA: y=0): The amount (in milligrams) of a drug in a person’s body following one dose is given by an exponential decay function. Let A(t) denote the amount of drug in the body at time t in hours after the dose was taken. In addition, suppose you know that A(3) 22.7 and A(6) 15.2. a. Find a formula for A in the form A(t) abt , where you determine the values of a 163 Chapter 3 Exponential and Logarithmic Functions and b exactly. b. What is the size of the initial dose the person was given? c. How much of the drug remains in the person’s body 8 hours after the dose was taken? d. Estimate how long it will take until there is less than 1 mg of the drug remaining in the body. e. Compute the average rate of change of A on the intervals [3, 5], [5, 7], and [7, 9]. Write at least one careful sentence to explain the meaning of the values you found, including appropriate units. Then write at least one additional sentence to explain any overall trend(s) you observe in the average rate of change. f. Plot A(t) on an appropriate interval and label important points and features of the graph to highlight graphical interpretations of your answers in (b), (c), (d), and (e)

APC: p. 165: Not a problem, but discover Newton’s Law of Cooling: if we have a cup of coffee at an initial temperature of 186◦ Fahrenheit and the cup is placed in a room where the surrounding temperature is 71◦ , our intuition and experience tell us that over time the coffee will cool and eventually tend to the 71◦ temperature of the surroundings. From an experiment ¹ with an actual temperature probe, we have the data in Table 3.2.1 that is plotted in Figure 3.2.2. t 0 1 2 3 8 13 F(t) 186 179 175 171 156 144 18 23 28 33 38 43 48 135 127 120 116 111 107 104 Table 3.2.1: Data for cooling coffee, measured in degrees Fahrenheit at time t in minutes. 10 20 30 40 40 80 120 160 t (min) F (degrees Fahrenheit) Figure 3.2.2: A plot of the data in Table 3.2.1

APC: p. 174: HW #4: A can of soda has been in a refrigerator for several days; the refrigerator has temperature 41◦ Fahrenheit. Upon removal, the soda is placed on a kitchen table in a room with surrounding temperature 72◦ . Let F(t) represent the soda’s temperature in degrees Fahrenheit at time t in minutes, where t 0 corresponds to the time the can is removed from the refrigerator. We know from Newton’s Law of Cooling that F has form F(t) abt + c for some constants a, b, and c, where 0 < b < 1. a. What is the numerical value of the soda’s initial temperature? What is the value of F(0) in terms of a, b, and c? What do these two observations tell us? b. What is the numerical value of the soda’s long-term temperature? What is the long-term value of F(t) in terms of a, b, and c? What do these two observations tell us? c. Using your work in (a) and (b), determine the numerical values of a and c. d. Suppose it can be determined that b 0.931. What is the soda’s temperature after 10 minutes?

OR (finding b):

APC: p. 174 HW #5: A cup of coffee has its temperature, C(t), measured in degrees Celsius. When poured outdoors on a cold morning, its temperature is C(0) 95. Ten minutes later, C(10) 80. If the surrounding temperature outside is 0 ◦ Celsius, find a formula for a function C(t) that models the coffee’s temperature at time t. In addition, recall that we can convert between Celsius and Fahrenheit according to the 175 Chapter 3 Exponential and Logarithmic Functions equations F 9 5 C+32 and C 5 9 (F−32). Use this information to also find a formula for F(t), the coffee’s Fahrenheit temperature at time t. What is similar and what is different regarding the functions C(t) and F(t)?

3-4-3 The special number e.

S-Z: p. 472: deriving e via more compoundings: We have observed that the more times you compound the interest per year, the more money you will earn in a year. Let’s push this notion to the limit.7 Consider an investment of $1 invested at 100% interest for 1 year compounded n times a year. Equation 6.2 tells us that the amount of money in the account after 1 year is A = 1 + 1 n n . Below is a table of values relating n and A. n A 1 2 2 2.25 4 ≈ 2.4414 12 ≈ 2.6130 360 ≈ 2.7145 1000 ≈ 2.7169 10000 ≈ 2.7181 100000 ≈ 2.7182 As promised, the more compoundings per year, the more money there is in the account, but we also observe that the increase in money is greatly diminishing. We are witnessing a mathematical ‘tug of war’. While we are compounding more times per year, and hence getting interest on our interest more often, the amount of time between compoundings is getting smaller and smaller, so there is less time to build up additional interest. With Calculus, we can show8 that as n → ∞, A = 1 + 1 n n → e, where e is the natural base first presented in Section 6.1. Taking the number of compoundings per year to infinity results in what is called continuously compounded inte

Calc-Medic: 3.3: 5. Is it even better to earn interest monthly or weekly? What about daily? Fill out the table to explore these options. Amount invested Interest is earned… Number of pay-outs per year % interest earned each pay-period Amount in account at end of year $1000 Monthly $1000 Weekly $1000 Daily 6. a) Do you think you could triple your money if you received payments multiple times a day? b) What is the most money you could have in your account after one year with the same initial investment of $1000?

S-Z p. 484 #20: 25(4/5)^x to 25e^(kx)

?Have equations: which ones require logarithms to solve?